Supplementary Information to:

Side reactions do not completely disrupt linear self-replicating chemical reaction systems

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- **Sum of reaction rate constants (Ω) linearly affects the growth rate (λ_1)**

Eq. (8) in the main text can be written in the form

$$\lambda^3 + X\Omega\lambda^2 + Y\Omega^2\lambda + Z\Omega^3 = 0$$

where $X = 1$, $Y = ab + ac + bc$ and $Z = -abc$. Based on the standard techniques to solve cubic equation, we first calculate:

$$\begin{aligned}
\Delta_0 &= (X\Omega)^2 - 3Y\Omega^2 \\
&= (X^2 - 3Y)\Omega^2 := \alpha\Omega^2 \\
\Delta_1 &= 2(X\Omega)^3 - 9(X\Omega)(Y\Omega^2) + 27Z\Omega^3 \\
&= (2X^3 - 9XY + 27Z)\Omega^3 := \beta\Omega^3 \\
H &= \sqrt[3]{\frac{\Delta_1 \pm \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}} \\
&= \Omega \cdot \sqrt[3]{\frac{\beta \pm \sqrt{\beta^2 - 4\alpha^3}}{2}} := \gamma\Omega
\end{aligned}$$

The three roots are

$$\begin{aligned}
\lambda'_k &= -\frac{1}{3}(X\Omega + \xi^k H + \frac{\Delta_0}{\xi^k H}) \\
&= -\frac{1}{3}(X + \xi^k\gamma + \frac{\alpha}{\xi^k\gamma})\Omega := Q_k \cdot \Omega
\end{aligned}$$

where $\xi = (-1 + \sqrt{3}j)/2$ and $k = 0, 1, 2$. So, Q_k is constant and only depends on X, Y and Z (equivalently, the relative rate constants, a, b and c). The maximum positive eigenvalue is thus

$$\lambda_1 = \max(\lambda'_0, \lambda'_1, \lambda'_2) = \Omega \cdot \max(Q_0, Q_1, Q_2)$$

Therefore, we proved that λ_1 is directly proportional to Ω.